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A Hybrid Method to Improve Forecasting Accuracy -Application to J-REIT (commerce, hotel, and logistics type) stock market prices

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Abstract

Given that the equation of the exponential smoothing method (ESM) is equivalent to the (1,1) order ARMA model equation, a new method of estimation of the smoothing constant in the exponential smoothing method was proposed before by us which satisfied the minimum variance of forecasting error. Generally, the smoothing constant is selected arbitrarily, but in this paper we utilize the above theoretical solution. Firstly, we estimate the ARMA model parameter and then estimate the smoothing constants. Thus the theoretical solution is derived in a simple way and may be utilized in various fields. Furthermore, combining the trend removing method with this method, we aim to improve forecasting accuracy. This new method is applied to the stock market price data of Japan Real Estate Investment Trust (J-REIT), obtaining some interesting results.

Keywords

minimum variance, exponential smoothing method, forecasting, trend

1 Introduction

Many methods for time series analysis have been presented such as Autoregressive model (AR Model), Autoregressive Moving Average Model (ARMA Model) and Exponential Smoothing Method (ESM) (Box Jenkins [1]), (R. G. Brown [11]), (Tokumaru et al. [3]), (Kobayashi [7]). Among these, ESM is said to be a practical simple method.

For this method, various improving method such as adding compensating item for time lag, coping with the time series with trend (Peter [10]), utilizing Kalman Filter (Maeda [4]), Bayes Forecasting (M. West et al. [8]), adaptive ESM (Steinar [13]), exponentially weighted Moving Averages with irregular updating periods (F.

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R. Johnston [2]), making averages of forecasts using plural method (Spyros [12]) are presented. For example, Maeda [4] calculated smoothing constant in relationship with S/N ratio under the assumption that the observation noise was added to the system. But he had to calculate under supposed noise because he couldn't grasp observation noise. It can be said that it doesn't pursue optimum solution from the very data themselves which should be derived by those estimation. Ishii [9] pointed out that the optimal smoothing constant was the solution of infinite order equation, but he didn't show analytical solution. Based on these facts, we proposed a new method of estimation of smoothing constant in ESM before (Takeyasu et al. [6]). Focusing that the equation of ESM is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in ESM was derived.

In this paper, utilizing above stated method, a revised forecasting method is proposed. In making forecast such as J-REIT stock market price data, trend removing method is devised. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function is executed to the original J-REIT stock market price data. The weights for these functions are varied by 0.01 increment and optimal weights are searched. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting is executed on these data. This is a revised forecasting method. Variance of forecasting error of this newly proposed method is assumed to be less than those of previously proposed method. The rest of the paper is organized as follows. In section 2, ESM is stated by ARMA model and estimation method of smoothing constant is derived using ARMA model identification. The combination of linear and non-linear function is introduced for trend removing in section 3. The Monthly Ratio is referred in section 4. Forecasting is executed in section 5, and estimation accuracy is examined.

2 Description of ESM Using ARMA Model (Takeyasu et al. [6])

In ESM, forecasting at time t+1 is stated in the following equation.

$$\begin{aligned} \hat{x}_{t+1} &= \hat{x}_t + \alpha\left(x_t - \hat{x}_t\right) \\ &= \alpha x_t + \left(1-\alpha\right)\hat{x}_t \end{aligned} \tag{1}$$

Here,

$\hat{x}_{t+1}$: forecasting at t+1

x_t : realized value at t

α : smoothing constant $\left(0 < \alpha < 1\right)$

(1) is re-stated as:

$$\hat{x}_{t+1} = \sum_{l=0}^{\infty} \alpha(1-\alpha)^l x_{t-l} \tag{2}$$

By the way, we consider the following (1,1) order ARMA model.

$$x_t - x_{t-1} = e_t - \beta e_{t-1} \tag{3}$$

Generally, (p, q) order ARMA model is stated as:

$$x_t + \sum_{i=1}^{p} a_i x_{t-i} = e_t + \sum_{j=1}^{q} b_j e_{t-j} \tag{4}$$

Here,

$\{x_t\}$: Sample process of Stationary Ergodic Gaussian Process $x(t)$ $t = 1,2,\cdots,N,\cdots$

$\{e_t\}$: Gaussian White Noise with 0 mean σ_e^2 variance

MA process in (4) is supposed to satisfy convertibility condition.

Utilizing the relation that:

$$E[e_t | e_{t-1}, e_{t-2}, \cdots] = 0$$

we get the following equation from (3).

$$\hat{x}_t = x_{t-1} - \beta e_{t-1} \tag{5}$$

Operating this scheme on $t+1$, we finally get:

$$\begin{aligned} \hat{x}_{t+1} &= \hat{x}_t + (1-\beta)e_t \\ &= \hat{x}_t + (1-\beta)(x_t - \hat{x}_t) \end{aligned} \tag{6}$$

If we set $1-\beta = \alpha$, the above equation is the same with (1), i.e., equation of ESM is equivalent to (1,1) order ARMA model, or is said to be (0,1,1) order ARIMA model because 1st order AR parameter is -1(Box Jenkins [1]), (Tokumaru et al. [3]).

Comparing with (3) and (4), we obtain:

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta \end{cases}$$

From (1), (6),

$$\alpha = 1 - \beta$$

Therefore, we get:

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \end{cases} \tag{7}$$

From above, we can get estimation of smoothing constant after we identify the parameter of MA part of ARMA model. But, generally MA part of ARMA model become non-linear equations which are described below.

Let (4) be:

$$\tilde{x}_t = x_t + \sum_{i=1}^{p} a_i x_{t-i} \tag{8}$$

$$\tilde{x}_t = e_t + \sum_{j=1}^{q} b_j e_{t-j} \tag{9}$$

We express the autocorrelation function of $\tilde{x}_t$ as $\tilde{r}_k$ and from (8), (9), we get the following non-linear equations which are well known [3].

$$\left.\begin{array}{l} \begin{cases} \tilde{r}_k = \sigma_e^2 \sum_{j=0}^{q-k} b_j b_{k+j} & (k \le q) \\ 0 & (k \ge q+1) \end{cases} \\ \tilde{r}_0 = \sigma_e^2 \sum_{j=0}^{q} b_j^2 \end{array}\right\} \tag{10}$$

For these equations, recursive algorithm has been developed. In this paper, parameter to be estimated is only b_1, so it can be solved in the following way.

From (3) (4) (7) (10), we get:

$$\left.\begin{array}{l} q = 1 \\ a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \\ \tilde{r}_0 = \left(1 + b_1^2\right)\sigma_e^2 \\ \tilde{r}_1 = b_1 \sigma_e^2 \end{array}\right\} \tag{11}$$

If we set:

$$\rho_k = \frac{\tilde{r}_k}{\tilde{r}_0} \tag{12}$$

the following equation is derived.

$$\rho_1 = \frac{b_1}{1 + b_1^2} \tag{13}$$

We can get b_1 as follows.

$$b_1 = \frac{1 \pm \sqrt{1 - 4\rho_1^2}}{2\rho_1} \tag{14}$$

In order to have real roots, ρ_1 must satisfy

$$|\rho_1| \le \frac{1}{2} \tag{15}$$

From invertibility condition, b_1 must satisfy

$$|b_1| < 1$$

From (13), using the next relation,

$$(1 - b_1)^2 \ge 0$$
$$(1 + b_1)^2 \ge 0$$

(15) always holds.

As

$$\alpha = b_1 + 1$$

b_1 is within the range of

$$-1 < b_1 < 0$$

Finally we get

$$\left.\begin{aligned} b_1 &= \frac{1-\sqrt{1-4\rho_1^2}}{2\rho_1} \\ \alpha &= \frac{1+2\rho_1-\sqrt{1-4\rho_1^2}}{2\rho_1} \end{aligned}\right\} \tag{16}$$

which satisfy above condition. Thus we can obtain a theoretical solution by a simple way.

Here ρ_1 must satisfy

$$-\frac{1}{2} < \rho_1 < 0 \tag{17}$$

in order to satisfy $0 < \alpha < 1$.

Focusing on the idea that the equation of ESM is equivalent to (1,1) order ARMA model equation, we can estimate smoothing constant after estimating ARMA model parameter.

It can be estimated only by calculating 0th and 1st order autocorrelation function.

3 Trend Removal Method

As trend removal method, we describe the combination of linear and non-linear function.

[1] Linear function

We set:

$$y = a_1 x + b_1 \tag{18}$$

as a linear function.

[2] Non-linear function

We set:

$$y = a_2 x^2 + b_2 x + c_2 \tag{19}$$

$$y = a_3 x^3 + b_3 x^2 + c_3 x + d_3 \tag{20}$$

as a 2nd and a 3rd order non-linear function.

[3] The combination of linear and non-linear function

We set:

$$\begin{aligned} y = {} & \alpha_1(a_1 x + b_1) + \alpha_2(a_2 x^2 + b_2 x + c_2) \\ & + \alpha_3(a_3 x^3 + b_3 x^2 + c_3 x + d_3) \end{aligned} \tag{21}$$

$$\begin{gathered} 0 \le \alpha_1 \le 1, \quad 0 \le \alpha_2 \le 1, \quad 0 \le \alpha_3 \le 1 \\ \alpha_1 + \alpha_2 + \alpha_3 = 1 \end{gathered} \tag{22}$$

as the combination of linear and 2nd order non-linear and 3rd order non-linear function. Trend is removed by dividing the data by (21). Numerical examples for about both of the trend removal case and the non-removal case are discussed in section 5.

4 Monthly Ratio

For example, if there is the monthly data of L years as stated bellow:

$$\{x_{ij}\}\ (i=1,\cdots,L)\ (j=1,\cdots,12)$$

where $x_{ij}\in R$ in which j means month and i means year and x_{ij} is a shipping data of i-th year, j-th month, then, monthly ratio $\tilde{x}_j\ (j=1,\cdots,12)$ is calculated as follows.

$$\tilde{x}_j=\frac{\frac{1}{L}\sum_{i=1}^{L}x_{ij}}{\frac{1}{L}\cdot\frac{1}{12}\sum_{i=1}^{L}\sum_{j=1}^{12}x_{ij}} \tag{23}$$

Monthly trend is removed by dividing the data by (23). Numerical examples both of the monthly trend removal case and the non-removal case are discussed in section 5.

5 Forecasting the Stock Market Price Data of J-REITs

5. 1 Analysis Procedure

Following five typical stocks are selected in which investment is a specialized type (Commerce, Hotel, and Logistics) rental field.

Japan Retail Fund Investment Corporation (“JRF”)
Frontier Real Estate Investment Corporation (“FRE”)
Japan Hotel and Resort, Inc. EIT, Inc. (“JHR”)
Nippon Hotel Fund Investment Corporation (“NHF”)
Japan Logistics Fund, Inc. (“JLF”)

Stock market price data are analyzed for the period of 2008 August to 2011 July. First of all, graphical charts of these time series data are exhibited in Figure 5-1 to 5-5.

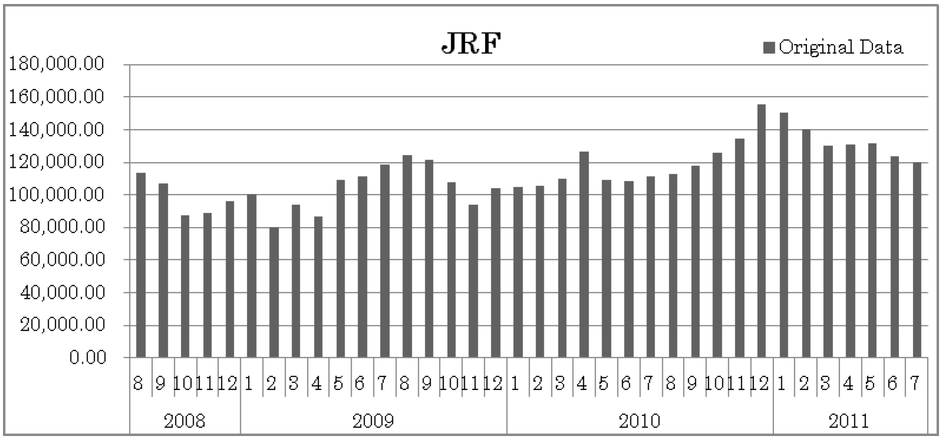

Fig. 5-1. Stock market price data of JRF

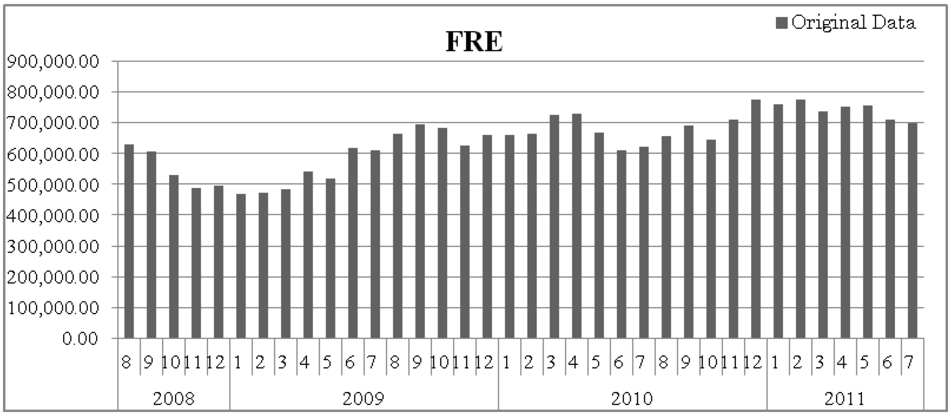

Fig. 5-2. Stock market price data of FRE

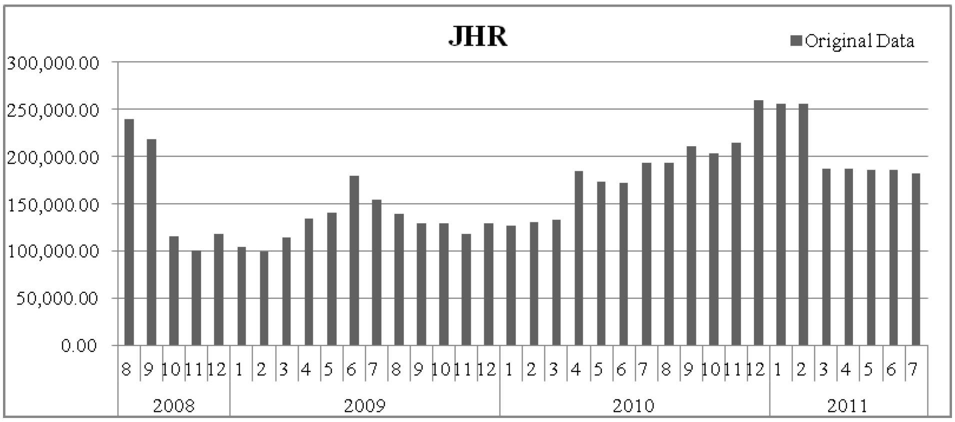

Fig. 5-3. Stock market price data of JHR

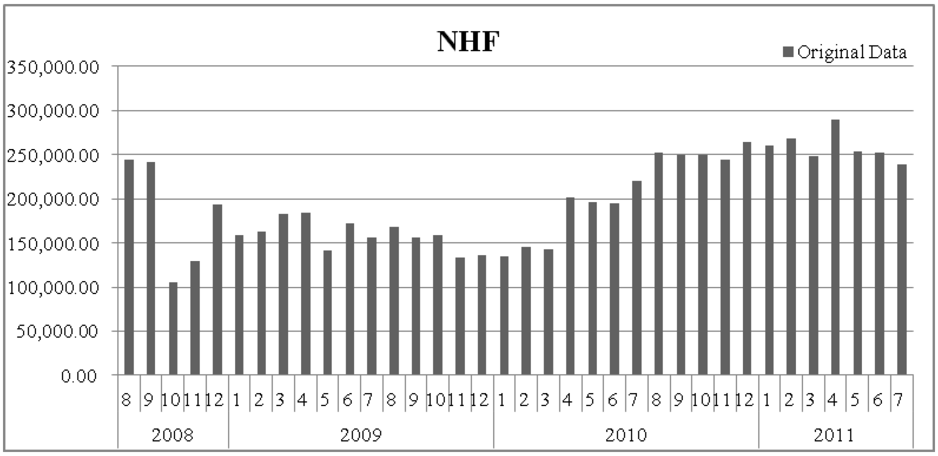

Fig. 5-4. Stock market price data of NHF

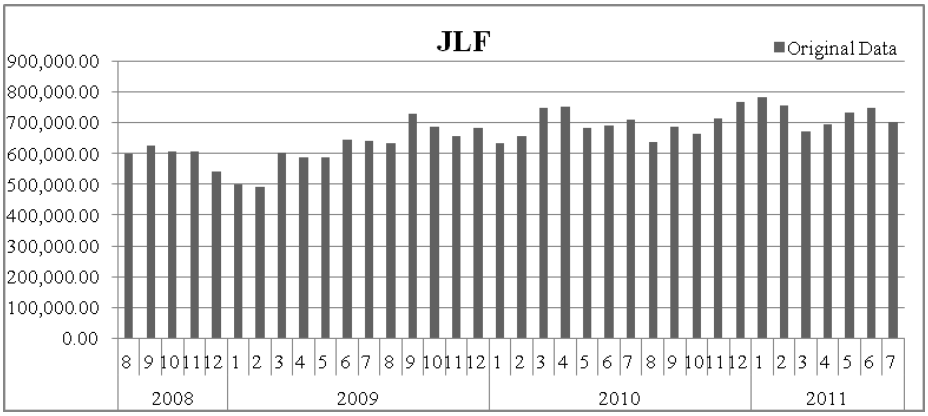

Fig. 5-5. Stock market price data of JLF

Analysis procedure is as follows. There are 36 monthly data for each case. We use 24 data(1 to 24) and remove trend by the method stated in section 3. Then we calculate monthly ratio by the method stated in section 4. After removing monthly trend, the method stated in section 2 is applied and Exponential Smoothing Constant with minimum variance of forecasting error is estimated. Then 1 step forecast is executed. Thus, data is shifted to 2nd to 25th and the forecast for 26th data is executed consecutively, which finally reaches forecast of 36th data. To examine the accuracy of forecasting, variance of forecasting error is calculated for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend.

Forecasting error is expressed as:

$$\varepsilon_i = \hat{x}_i - x_i \tag{24}$$

$$\bar{\varepsilon} = \frac{1}{N}\sum_{i=1}^{N} \varepsilon_i \tag{25}$$

Variance of forecasting error is calculated by:

$$\sigma_\varepsilon^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(\varepsilon_i - \bar{\varepsilon}\right)^2 \tag{26}$$

Then the Standard Deviation of forecasting error is calculated by:

$$\sigma_\varepsilon = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\varepsilon_i - \bar{\varepsilon}\right)^2} \tag{27}$$

Comparison Index (CI) is introduced as follows.

$$CI = \frac{\sigma_\varepsilon}{\bar{x}} \tag{28}$$

Where $\bar{x}$ is stated as:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{29}$$

In this paper, we examine the four cases stated in Table 5-1.

Table 5-1. The combination of the case of trend removal and monthly trend removal

	Trend	Monthly trend
Case1	Removal	Removal
Case2	Removal	Non removal
Case3	Non removal	Removal
Case4	Non removal	Non removal

5. 2 Trend Removing

Trend is removed by dividing the original data by (21). Here, the weight of α_1 and α_2 are shifted by 0.01 increment in (21) which satisfy the equation (22). The best solution is selected which minimizes the variance of forecasting error. Estimation results of coefficient of (18), (19) and (20) are exhibited in Table 5-2. Data are fitted to (18), (19) and (20), and using the least square method, parameters of (18), (19) and (20) are estimated. Estimation results of weights of (21) are exhibited in Table 5-3. The weighting parameters are selected so as to minimize the variance of forecasting error.

Table 5-2. Coefficient of (18),(19) and (20)

	1st		2nd			3rd			
	a_1	b_1	a_2	b_2	c_2	a_3	b_3	c_3	d_3
JRF	752	95,671	3	685	95,961	-14	534	-4,740	108,399
FRE	7,795	505,351	33	6,975	508,904	-194	7,318	-67,313	679,380
JHR	483	1,388	494	-11,870	192,364	-34	1,771	-24,905	222,250
NHF	-250	172,391	452	-11,544	221,333	12	-1	-6,924	210,741
JLF	7,201	547,192	175	2,828	566,144	-90	3,563	-31,746	645,419

Table 5-3. Weights of (21)

		α_1	α_2	α_3
JRF	Case1	0.61	0.00	0.39
	Case2	1.00	0.00	0.00
FRE	Case1	0.56	0.00	0.44
	Case2	0.99	0.00	0.01
JHR	Case1	0.33	0.05	0.62
	Case2	1.00	0.00	0.00
NHF	Case1	0.00	0.37	0.63
	Case2	0.46	0.29	0.25
JLF	Case1	0.50	0.00	0.50
	Case2	1.00	0.00	0.00

As a result, we can observe the following four patterns.

①Selected liner model:

JRF: Case2, JHR: Case2, JLF: Case2

②Selected 1st+3rd order model:

JRF: Case1, FRE: Case1, Case2, JLF: Case1

③Selected 2nd+3rd order model:

NHF: Case1

④Selected 1st+2nd+3rd order model:

JHR: Case1, NHF: Case2

Graphical charts of trend are exhibited in Figure 5-6 to 5-10.

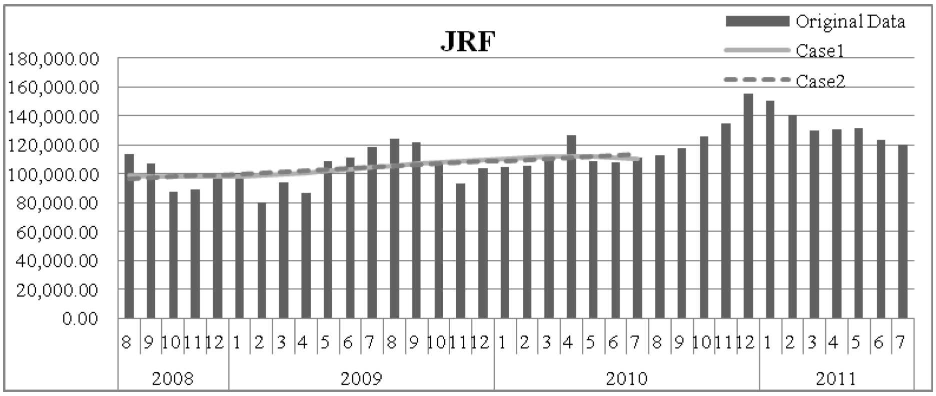

Fig. 5-6. Trend of JRF

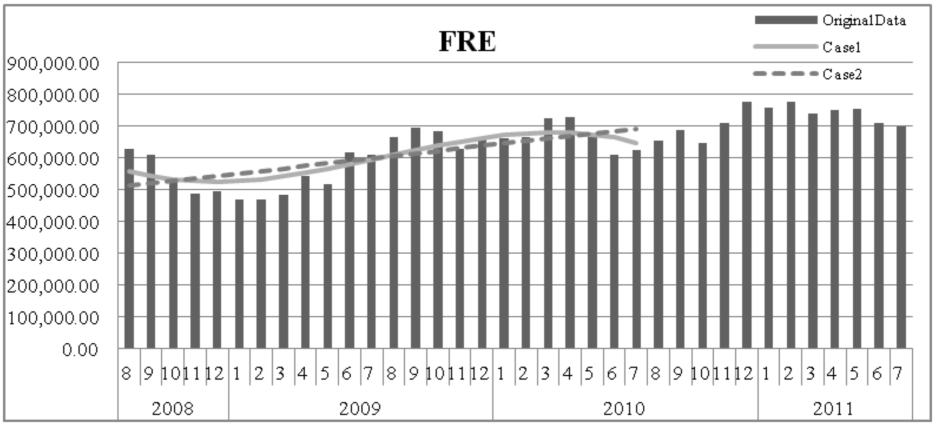

Fig. 5-7. Trend of FRE

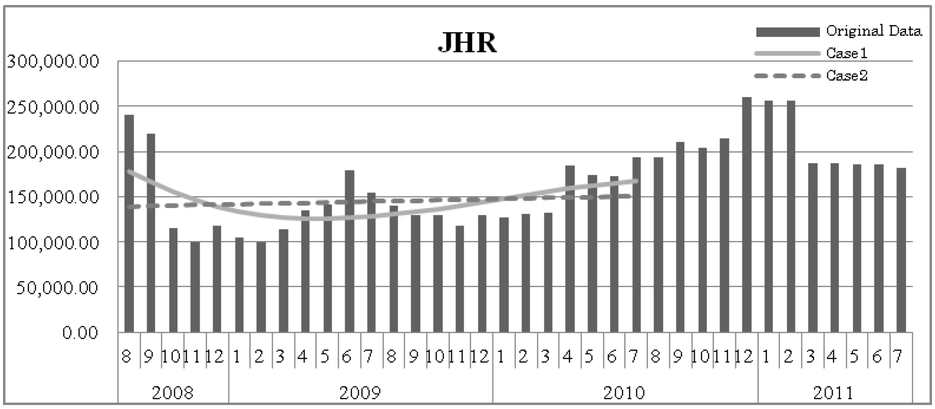

Fig. 5-8. Trend of JHR

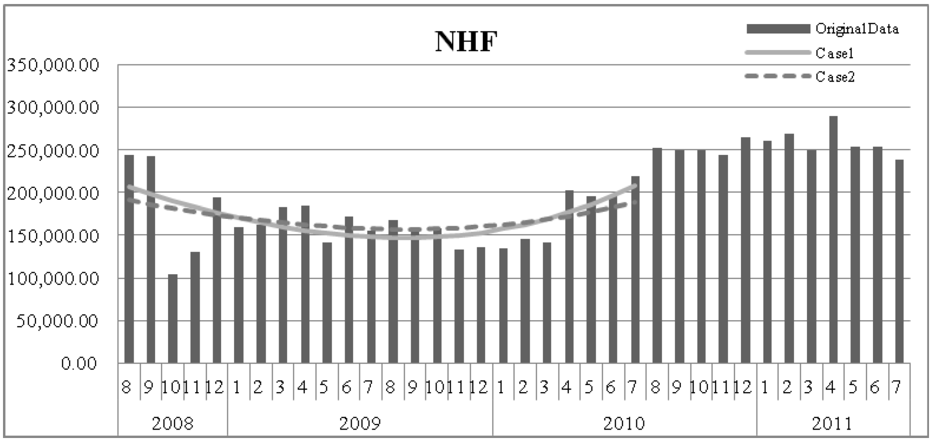

Fig. 5-9. Trend of NHF

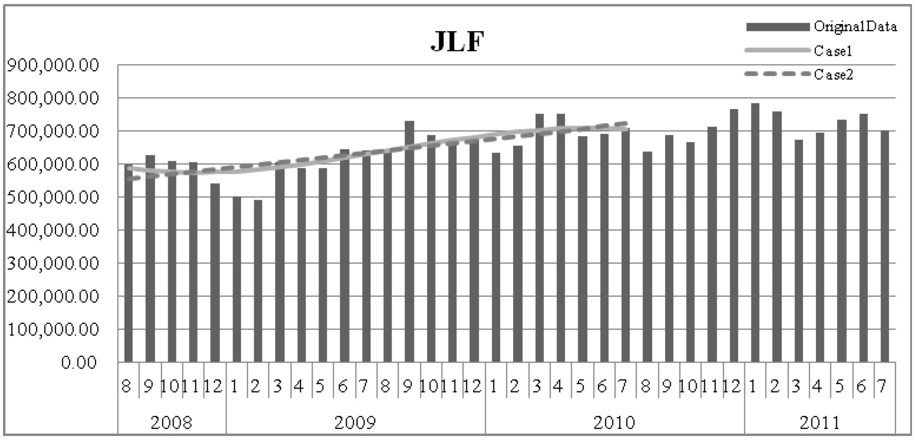

Fig. 5-10. Trend of JLF

5. 3 Removing Trend of Monthly Ratio

After removing trend, monthly ratio is calculated by the method stated in 4. Calculation result for 1st to 24th data is exhibited in Table 5-4.

Table 5-4. Monthly ratio

		Month											
		8	9	10	11	12	1	2	3	4	5	6	7
JRF	Case1	1.16	1.11	0.95	0.89	0.96	0.98	0.88	0.96	0.99	1.02	1.03	1.07
	Case3	1.13	1.09	0.93	0.87	0.95	0.98	0.88	0.97	1.01	1.04	1.05	1.10
FRE	Case1	1.11	1.12	1.04	0.95	0.97	0.94	0.93	0.98	1.03	0.95	0.99	1.00
	Case3	1.07	1.08	1.01	0.92	0.96	0.94	0.94	1.00	1.05	0.98	1.02	1.02
JHR	Case1	1.21	1.15	0.85	0.77	0.88	0.82	0.81	0.88	1.12	1.10	1.24	1.18
	Case3	1.31	1.20	0.84	0.75	0.85	0.80	0.79	0.85	1.10	1.09	1.21	1.20
NHF	Case1	1.16	1.14	0.81	0.80	0.99	0.90	0.94	0.99	1.16	0.99	1.07	1.05
	Case3	1.22	1.18	0.78	0.78	0.97	0.87	0.91	0.96	1.14	1.00	1.09	1.11
JLF	Case1	1.01	1.10	1.05	1.02	0.97	0.89	0.89	1.05	1.03	0.97	1.01	1.01
	Case3	0.97	1.06	1.02	0.99	0.96	0.89	0.90	1.06	1.05	1.00	1.05	1.06

5. 4 Estimation of Smoothing Constant with Minimum Variance of Forecasting Error

After removing monthly trend, Smoothing Constant with minimum variance of forecasting error is estimated utilizing (16). There are cases that we cannot obtain a theoretical solution because they do not satisfy the condition of (17). In those cases, Smoothing Constant with minimum variance of forecasting error is derived by shifting variable from 0.01 to 0.99 with 0.01 interval. Calculation result for 1st to 24th data is exhibited in Table 5-5.

Table 5-5. Estimated Smoothing Constant with Minimum Variance

		ρ_1	α
JRF	Case1	**-0.7940(Does not satisfy (17))**	**0.9900**
	Case2	-0.1707	0.8240
	Case3	-0.4926	0.1587
	Case4	-0.1760	0.8182
FRE	Case1	-0.2232	0.7644
	Case2	**-0.5500(Does not satisfy (17))**	**0.1700**
	Case3	**-0.6486(Does not satisfy (17))**	**0.9900**
	Case4	**-0.52614(Does not satisfy (17))**	**0.8200**
JHR	Case1	-0.1684	0.8266
	Case2	-0.4238	0.4463
	Case3	-0.0185	0.9815
	Case4	-0.4280	0.4357
NHF	Case1	-0.2999	0.6667
	Case2	-0.2607	0.7187
	Case3	-0.2670	0.7107
	Case4	-0.2037	0.7871

JLF	Case1	-0.3132	0.6480
	Case2	-0.0882	0.9111
	Case3	-0.0833	0.9161
	Case4	-0.0982	0.9008

5. 5 Forecasting and Variance of Forecasting Error

Utilizing smoothing constant estimated in the previous section, forecasting is executed for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Variance of forecasting error is calculated by (26). Forecasting results are exhibited in Figure 5-11 to 5-15 for the cases that monthly ratio is used.

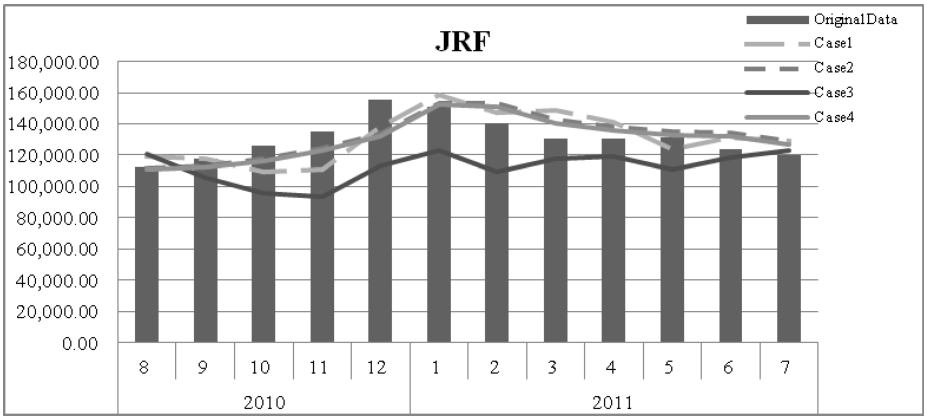

Fig. 5-11. Forecasting Results of JRF

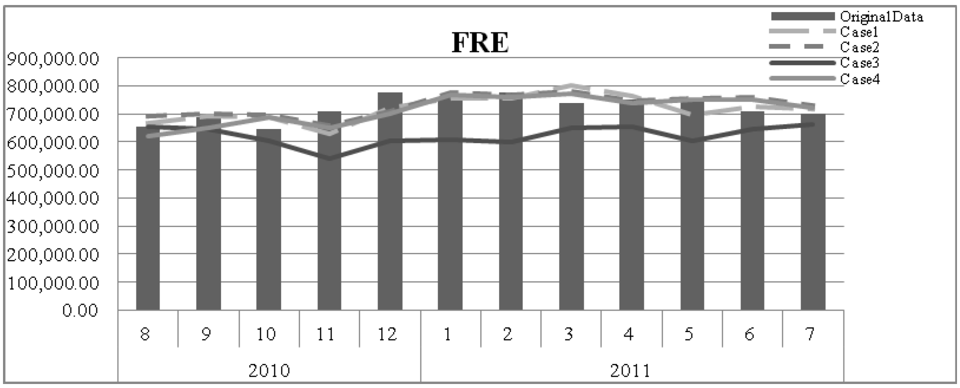

Fig. 5-12. Forecasting Results of FRE

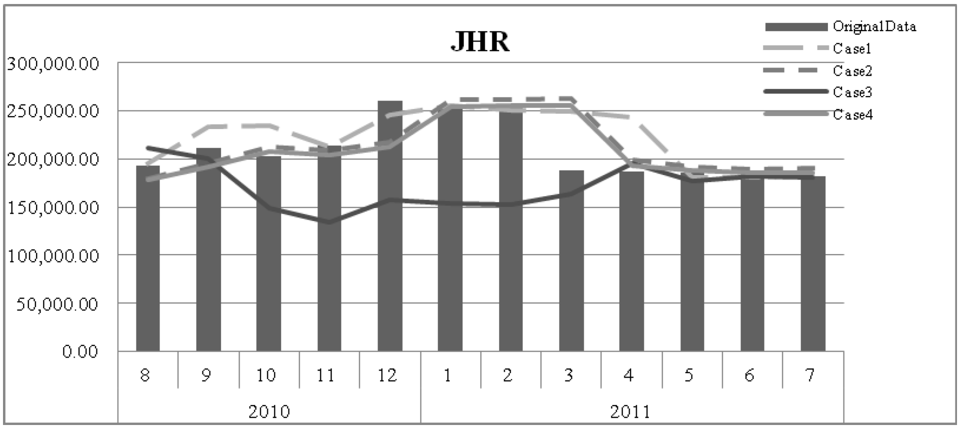

Fig. 5-13. Forecasting Results of JHR

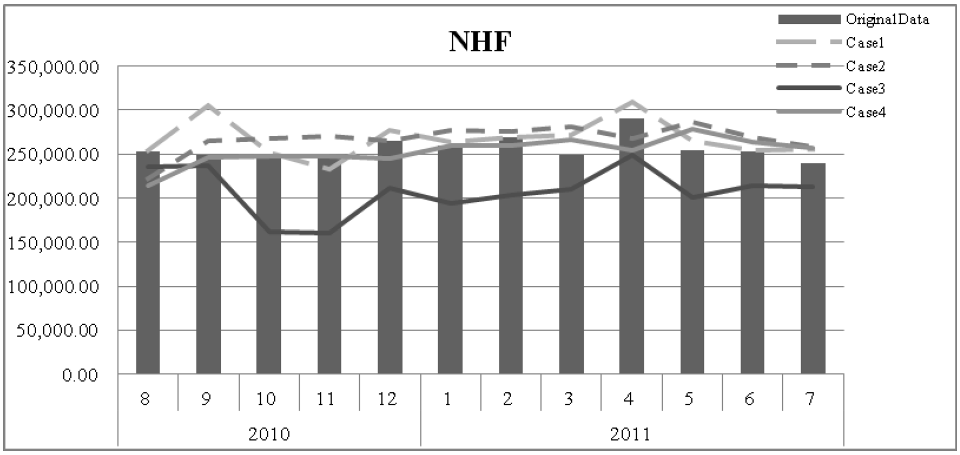

Fig. 5-14. Forecasting Results of NHF

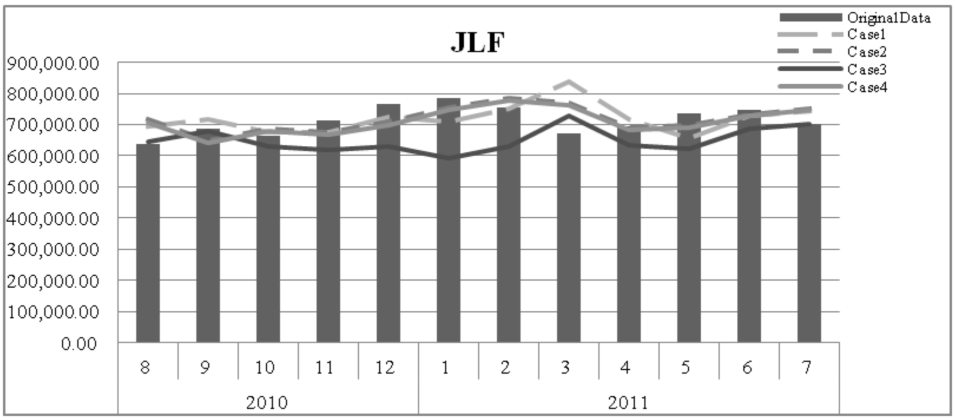

Fig. 5-15. Forecasting Results of JLF

Variance of forecasting error is exhibited in Table 5-6.

Table 5-6. Variance of Forecasting Error

		Variance of Forecasting Error	
JRF	Case1	178,898,373	
	Case2	113,975,424	
	Case3	263,385,772	
	Case4	106,001,262	*
FRE	Case1	1,821,530,521	
	Case2	1,413,882,202	*
	Case3	3,893,229,140	
	Case4	1,570,845,445	
JHR	Case1	647,927,669	*
	Case2	737,379,873	
	Case3	2,182,798,385	
	Case4	698,955,352	
NHF	Case1	291,586,725	*
	Case2	387,542,415	
	Case3	592,298,095	
	Case4	402,857,569	
JLF	Case1	4,463,309,877	
	Case2	2,525,521,061	*
	Case3	5,181,487,676	
	Case4	2,528,229,802	

In order to compare the forecasting accuracy by the difference of the investment portfolio, similar calculations are performed about other “Office type”, “Residential type” and “Complex type” J-REIT by the data of the same period.

Following five typical stocks are selected in which investment is a specialized office type rental field.

Nippon Building Fund Inc. (“NBF”)

Japan Real Estate Investment Corporation (“JRE”)

Global One Real Estate Investment Corporation (“GOR”)

Nomura Real Estate Office Fund, Inc. (“NRE”)

Daiwa Office Investment Corporation (“DOI”)

Following three typical stocks are selected in which investment is a residential rental field.

Japan Rental Housing Investments Inc. (“JRH”)

Nippon Accommodations Fund (“NAF”)

NOMURA Residential Fund ("NRF")

And following five typical stocks are selected in which investment is a complex type (office and others) rental field.

Japan Prime Realty Investment Corporation ("JPR")
Premier Investment Corporation ("PIC")
TOKYU REIT, Inc. ("TRI")
HEIWA REAL ESTATE REIT, Inc.("HRE")
Ichigo Real Estate Investment Corporation ("IRE")

Comparison Indices (CI: Equation (28)) are exhibited in Fig.5-16 to 5-19.

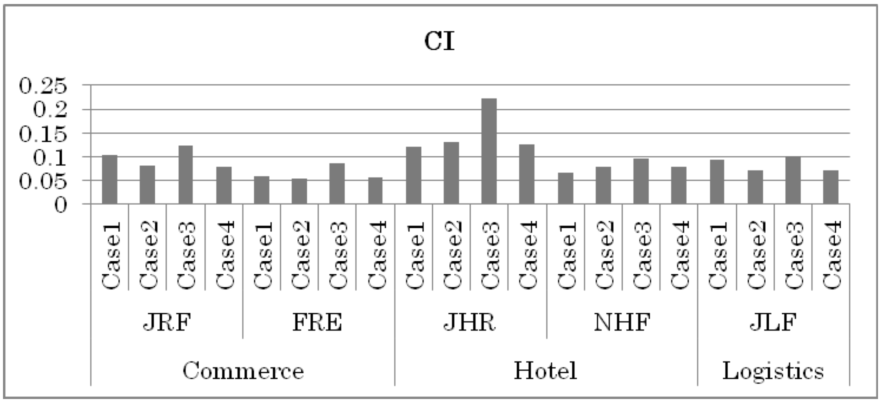

Fig. 5-16. Comparison Index about Commerce, Hotel and Logistics Type

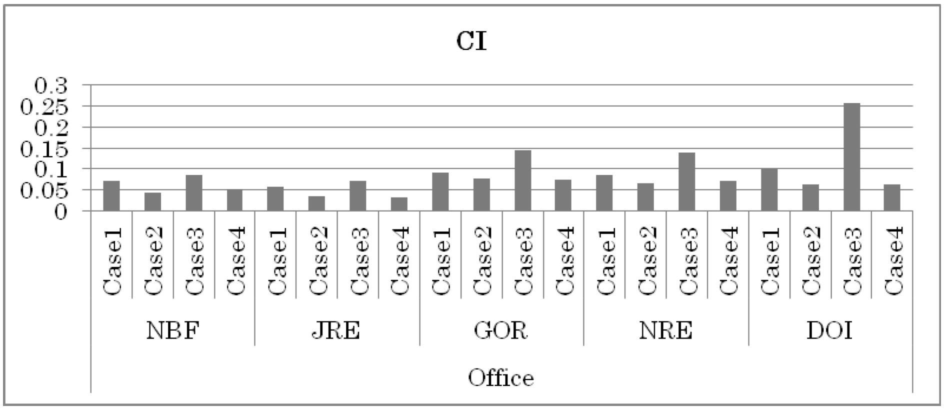

Fig. 5-17. Comparison Index about Office Type

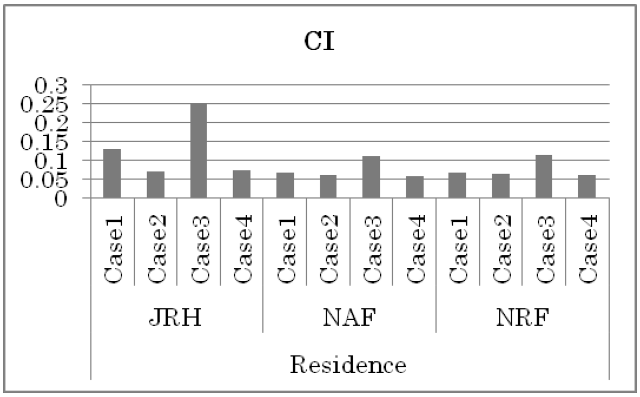

Fig. 5-18. Comparison Index about Residence Type

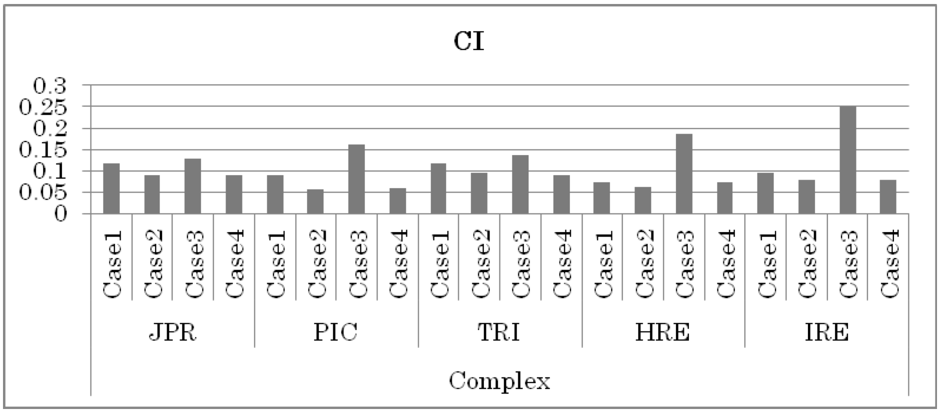

Fig. 5-19. Comparison Index about Complex Type

5. 6 Remarks

Case1 and Case3 where monthly trend removal are executed had bad forecasting results in these cases. In particular, Case3 where monthly trend removal and trend non-removal are executed had worst forecasting results in all cases. It may be because there was a big down shift according to the influence of sub-prime loan crisis and Lehman Shock occurred at 2008. Therefore monthly trend removal is not inevitable in these cases.

And we could not find the difference of the forecasting accuracy by the difference of the investment portfolio of the corporations. It seems that the difference of the forecasting accuracy came out by the individual circumstances of the corporations rather than the difference of the investment portfolio of them. We can see that the corporations whose forecasting accuracy was not good compared with the other corporations, are the corporations strongly influenced by the Riemann shock especially.

6 Conclusion

Focusing on the idea that the equation of exponential smoothing method (ESM) was equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was proposed before by us which satisfied minimum variance of forecasting error. Generally, smoothing constant was selected arbitrarily. But in this paper, we utilized above stated theoretical solution. Firstly, we made estimation of ARMA model parameter and then estimated smoothing constants. Thus theoretical solution was derived in a simple way and it might be utilized in various fields.

Furthermore, combining the trend removal method with this method, we aimed to improve forecasting accuracy. An approach to this method was executed in the following method. Trend removal by a linear function was applied to the stock market price data of J-REIT. The combination of linear and non-linear function was also introduced in trend removing. For the comparison, monthly trend was removed after that. Theoretical solution of smoothing constant of ESM was calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting was executed on these data.

If the data have a big trend, trend removing is inevitable. This was verified by the plural case studies. Various cases should be examined hereafter.

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